Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 7 June 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 6 5 0 8 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $f(x) = 6x^3 - 13x^2 + ax - 10$ where a is a constant

Given that $(3x - 2)$ is a factor of $f(x)$

(a) show that $a = 21$

(2)

(b) Hence show algebraically that the curve $y = f(x)$ has only one intersection with the x-axis.

(4)

(Total for Question 1 is 6 marks)

2 The quadratic equation $3x^2 - 5x + 1 = 0$ has roots α and β

Without solving the equation,

form a quadratic equation with integer coefficients, that has roots $\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$ **(8)**

Question 2 continued

(Total for Question 2 is 8 marks)

3

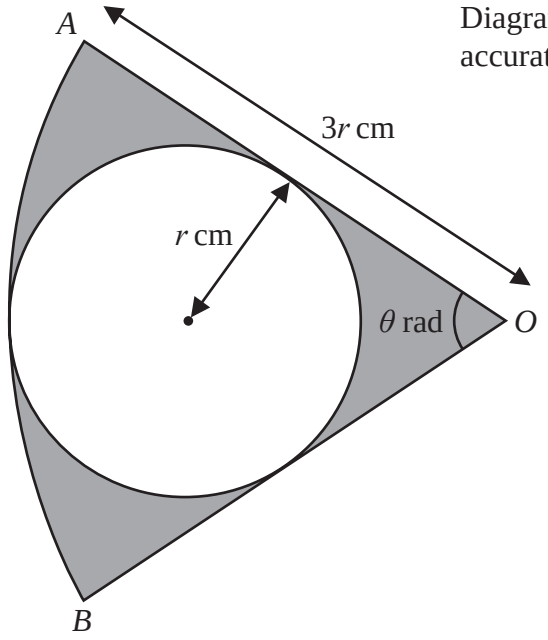

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius $3r$ cm

A circle with radius r cm touches OA and OB and the arc AB

Angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of θ

(2)

The area of the region shown shaded in Figure 1 is 8π cm^2

(b) Find the value of r

(4)

Question 3 continued

(Total for Question 3 is 6 marks)

4

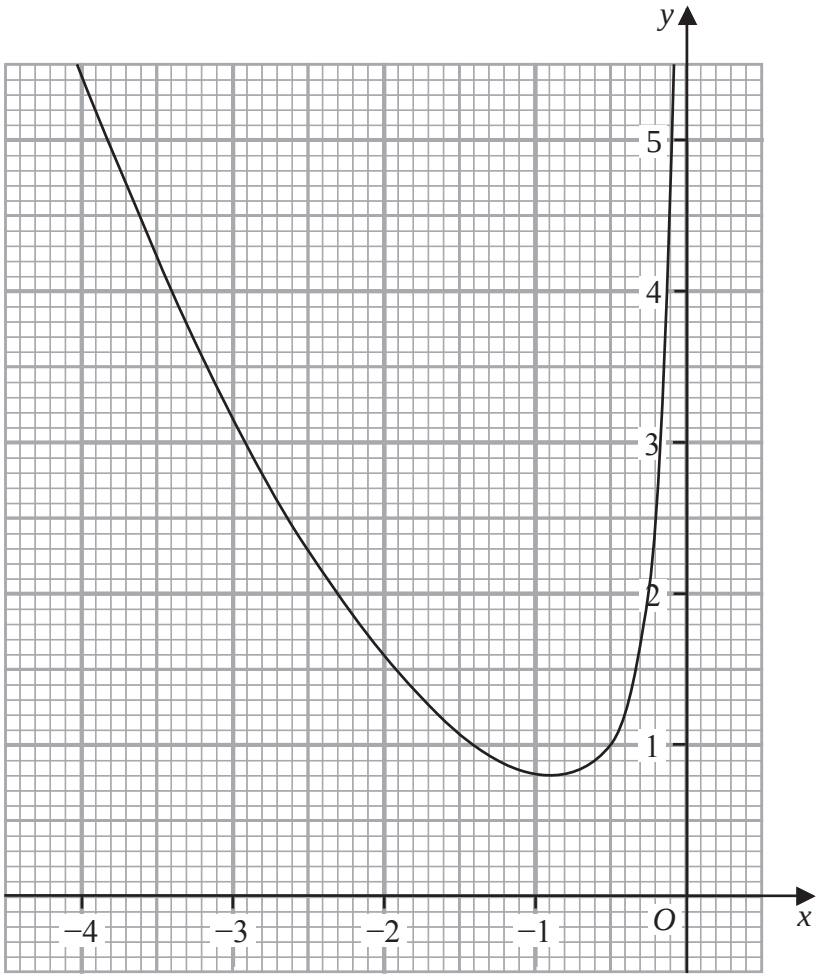

Figure 2

Figure 2 shows part of the curve with equation $y = \frac{x^2}{3} - \frac{1}{2x}$ for $-4 < x < 0$

By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 + 3x^2 - 36x - 6 = 0$ in the interval $-4 < x < 0$

(4)

Question 4 continued

(Total for Question 4 is 4 marks)

5

$$y = e^{2x}\left(x^2 - 5x\right)$$

Show that $2e^{2x} = \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y$

(7)

Question 5 continued

(Total for Question 5 is 7 marks)

6

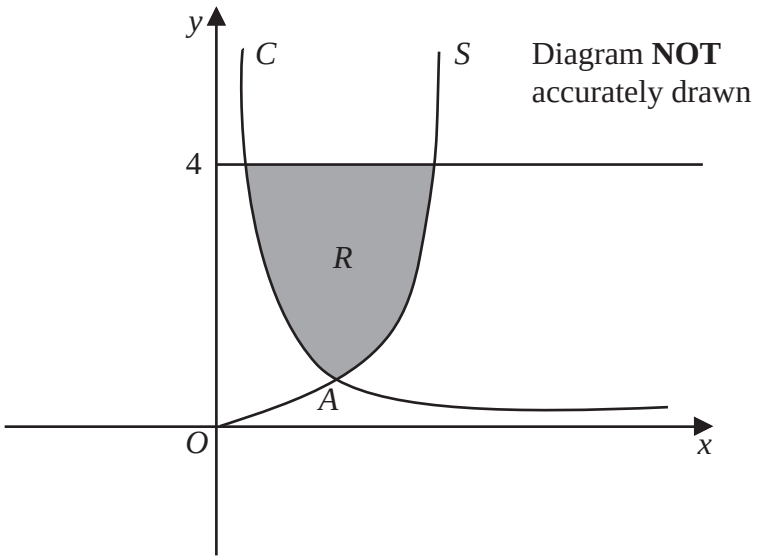

Figure 3

Figure 3 shows part of the curve C with equation $y = \frac{1}{4x}$, $x > 0$ and part of the curve S with equation $y = 2x^2, x \geqslant 0$

The curve C and the curve S intersect at the point A

(a) Find the coordinates of point A

(3)

The finite region R, shown shaded in Figure 3, bounded by the curve C, the curve S and the straight line $y = 4$ is rotated through 360° about the y-axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

(7)

Question 6 continued

(Total for Question 6 is 10 marks)

7 (a) Expand $\left(1+2x^2\right)^{-\frac{3}{4}}$ in ascending powers of x up to and including the term in x^6

Express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{(2+kx)}{\left(1+2x^2\right)^{\frac{3}{4}}} \quad \text{where } k \neq 0$$

(b) Obtain a series expansion for f(x) in ascending powers of x up to and including the term in x^5

Give each coefficient in terms of k where appropriate.

(2)

The coefficient of the term in x^5 is fourteen times the coefficient of the term in x^2

(c) Find the value of k

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8

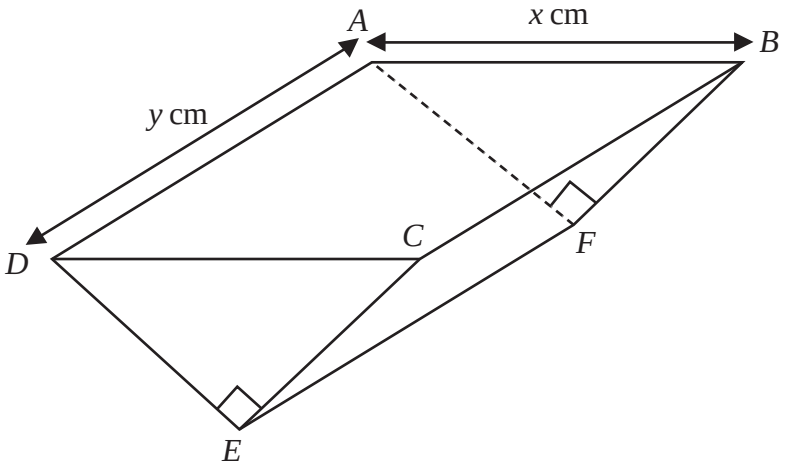

Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a solid right triangular prism *ABCDEF*

The cross section of the prism is an isosceles triangle.

- $\angle DEC = \angle AFB = 90^{\circ}$
- $AB = DC = x$ cm
- $AD = BC = FE = y$ cm
- $AF = BF = DE = CE$

The triangular faces of the prism are vertical and the edges *AD*, *BC* and *FE* are horizontal.
The volume of the prism is 3.6 cm^3
The total external surface area of the prism is *S* cm^2

(a) Show that *S* satisfies the equation

$$S = \frac{x^2}{2} + \frac{72\left(\sqrt{2}+1\right)}{5x}$$

(4)

Given that *x* can vary,

(b) use calculus, to find to 3 significant figures, the value of *x* for which *S* is a minimum.
Justify that this value of *x* gives a minimum value of *S*

(4)

(c) Hence find, to 2 significant figures, the minimum value of *S*

(2)

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.....

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9

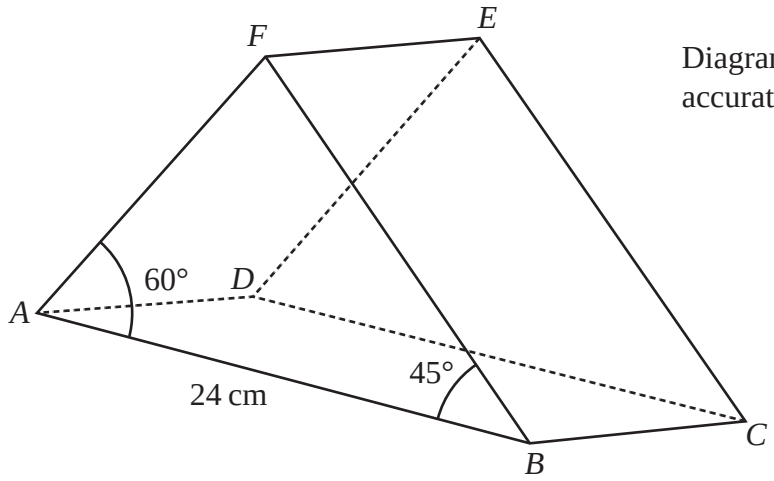

Diagram **NOT** accurately drawn

Figure 5

Figure 5 shows a right triangular prism *ABCDEF* where *ABCD* is a rectangle.

$AF = DE$ $BF = CE$ $AD = FE = BC$ $AB = DC = 24$ cm

$\angle ABF = \angle DCE = 45°$ $\angle BAF = \angle CDE = 60°$

Using a formula from page 2,

(a) show that $\sin AFB = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

(3)

Without using a calculator,

(b) show that $BF = 12\left(3\sqrt{2} - \sqrt{6}\right)$ cm

(5)

The angle between the plane *AEB* and the plane *ABCD* is 65°

(c) Find, in cm to 2 significant figures, the length of *EF*

(3)

(d) Find, in degrees to one decimal place, the size of the angle between the line *CF* and the plane *ABCD*

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 15 marks)

10 The points *A*, *B*, *C* and *D* are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b} \quad \overrightarrow{AC} = 7\mathbf{a} + 9\mathbf{b} \quad \overrightarrow{AD} = 4\mathbf{a} + 5\mathbf{b}$$

(a) Show that *ABCD* is a parallelogram.

(3)

BC is extended to the point *E* such that *BCE* is a straight line.

Point *F* lies on *CD* such that $CF : FD = 1 : 2$

Given that *A*, *F* and *E* are collinear,

(b) find the vector $\overrightarrow{AE}$ in the form $X\mathbf{a} + Y\mathbf{b}$ where *X* and *Y* are rational numbers to be found.

(8)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

11 Using formulae from page 2, show that

(a) (i) $\cos 2A = 2\cos^2 A - 1$

(3)

(ii) $\sin 2A = 2\sin A\cos A$

(1)

(b) Show that $\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$

(4)

Hence, or otherwise,

(c) solve, giving exact values in terms of π

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \qquad \text{for } 0 \leqslant \theta \leqslant 2\pi$$

(4)

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(4\cos^3\theta - \sin 2\theta\right) \mathrm{d}\theta$$

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS